CHAPTER II
PROSPECTS OF HEAVY-ION COLLISIONS

The ultimate goal for nuclear physicists is to understand QCD matter and its governing Equation of State (EoS). In order to study this strongly interacting matter, we need sufficient energy to tear the nucleus apart, generating conditions where the “strong force” is dominant. Hence, we need heavy-ion collisions. In this chapter, we introduce the concepts and goals of heavy-ion collisions.

2.1 Exploring the QCD Phase Diagram

2.1.1 The Development of Models and Equation of State

Early theoretical prediction for a new state of nuclear matter was done in the 1970s by Ref. (Fritzsch et al., 1973; Freedman and McLerran, 1977; Shuryak, 1978; McLerran, 1986) regarding nuclear matter under extreme conditions leading to the so-called “Quark Gluon Plasma (QGP)”. Naturally, one expects an accompanying phase transition between two different states in the phase diagram. A type of transition is dictated by the order parameter or a relation between thermodynamic properties, e.g., pressure, energy density, chemical potential, and temperature. This relation is known as “Equation of State (EoS)”. Ref. (Cabibbo and Parisi, 1975) reported that there could be many kinds of orders of phase transitions in the QCD phase diagram. A smooth second-order phase transition or a crossover at high temperatures with low baryochemical potential is expected from the earlier theoretical calculation either by chiral model or lattice QCD (see below). Then there should be a first-order phase transition line with a critical end point where the thermodynamic changes are abruptly strong and even diverge. However, since then, there is still no clear experimental evidence to prove the existence of this critical point.

During this period, Ref. (Johnson, 1975) has proposed an empirical approach to explain the physical behaviors that could provide the EoS with the first-order phase transition as well as a critical end point. This model is called the MIT Bag Model.

Only until 1990s, the first principle non-perturbative QCD calculation could be successfully published. The approach is known as “lattice QCD”. It is based on the discretizing the space-time grid, allowing for numerical simulation processes (Karsch, 2002; Fodor and Katz, 2002). After a decade, this approach has become one of the most basis for the simulation comparison due to its fundamental nature. The development of the lattice QCD is now focusing on extending its computational ability via complex numerical techniques for simulating the QCD matter at the larger chemical potential. Currently, the lattice QCD could provide the reliable EoS and confirm the smooth crossover phase transition at the region $\mu_B/T \leq \pi$ (no critical point has been found) (Allton et al., 2005; Aoki et al., 2006; Vovchenko et al., 2018b).

Another important theoretical development was the introduction of the chiral models (Brown and Rho, 1996; Berges and Rajagopal, 1999; Alford et al., 1999). The chiral model could also provide a similar physical description of the two major phases, the chiral symmetry restored phase and the chiral spontaneously breaking phase, which is comparable to the de-confinement phase at QGP conditions and the confinement phase at low temperature and baryochemical potential where quarks are bound into normal hadrons, respectively. Despite arising from the same origin, some simple chiral models provide only a first-order phase transition (Koch, 1997), while some models could suggest a critical end point. However, the specific details remain under investigation.

Aside from the theoretical developments, the advancement of heavy-ion collision facilities also plays an important role in nuclear physics. The very first announcement for the existence of QGP was presented by CERN Super Proton Synchrotron (SPS) with several collaborations in the 1990s (Heinz and Jacob, 2000). Since the first heavy-ion collision experiments from Bevalac to the Large Hadron Collider (LHC) at CERN, the data collected over the years are immersed and further confirms the existence of the QGP definitively with a multitude of observables, such as strangeness enhancement K/π, the J/Ψ suppression (Matsui and Satz, 1986; Wiedemann, 2010), the collective flow (Muller et al., 2012).

Despite massive evidence of the new phase of nuclear matter QGP, the exploration of the QCD phase diagram still remains being an active research (Collaboration, 2014) in order to study the physical behavior of the phase transition and locate the critical end point (if exists). Over the past decades with theoretical and experimental developments, there seem to be more than two phases of QCD matters, e.g., Color-Glass superconductor (CGC) phase at low temperature but very high baryochemical potential, as well as the connection between nuclear physics and the very dense/compact stellar objects, like neutron stars, i.e., the EoS. Thus, future efforts will likely focus on the lower energies but extremely dense conditions for extracting the EoS as well as studying the critical behavior of the QCD matter undergoing the phase transition (as well as the critical point).

The equation of state describes the relation between the thermodynamic quantities. These quantities can be obtained from the partition function of any equilibrium system parametrized by the volume V, temperature T, and chemical potential μ. For instance, in the grand-canonical picture of monotonic gas we have,

$$Z(T, V, \mu) = \mathrm{Tr}\left[\exp^{-(\hat{H}-\mu\hat{N})/T}\right] , \tag{2.1}$$

$$\Omega(T, V, \mu) = -T\ln Z . \tag{2.2}$$

$Z(T, V, \mu)$ is the grand-canonical partition function with $\Omega(T, V, \mu)$ as a grand potential. Here $\hat{H}$ and $\hat{N}$ are the Hamiltonian operator and the number operator. The thermodynamic relations is

$$\Omega(T, V, \mu) = E - TS - \mu N , \tag{2.3}$$

$$d\Omega(T, V, \mu) = -SdT - pdV - Nd\mu , \tag{2.4}$$

where S is the average entropy. Oftentimes, the thermodynamic quantities are expressed in terms of density,

$$-P = \epsilon - Ts - \mu n , \tag{2.5}$$

$$dP = sdT + nd\mu , \tag{2.6}$$

$$d\epsilon = Tds + \mu dn . \tag{2.7}$$

The following relations are also useful:

$$\epsilon = \frac{T}{V}\left(\frac{\partial \ln Z}{\partial \ln T} + \frac{\partial \ln Z}{\partial \ln \mu}\right) , \tag{2.8}$$

$$P = T\frac{\partial \ln Z}{\partial V} . \tag{2.9}$$

We now have all of the connections required to create the equation of state.

Non-interacting relativistic particles: The grand-canonical partition function for non-interacting massive particles is given by,

$$Z(T, V, \mu) = \prod_k \left(\sum_\sigma^\infty e^{-\sigma(E(k)-\mu)/T} \right)^d = \prod_k \left(1 \pm e^{-(E(k)-\mu)/T} \right)^{\pm d} .$$

The partition function is expressed as the production of a single momentum state, which is a sum over all possible same occupying states, i.e., the occupation number σ. For bosons, they can take any state up to the infinite number $\sigma = 0, 1, 2, 3, \ldots$, while fermions can only take the same state up to $\sigma = 0, 1$ due to Pauli's exclusion. The d is the degeneracy factor accounting for all possible spin states.

In an extreme case like the early universe QGP, we expect a very high temperature with $\mu \to 0$. Under these conditions, the system is approximately at the chiral limit, where the bare quarks are massless. The system can be described with massless Goldstone boson gas. The grand potential density or pressure of massless boson gas reads as,

$$P = \begin{cases} \text{for bosons;} \\ d \int \frac{d^3k}{(2\pi^3)} T \ln \left(1 - e^{-E(k)/T} \right) = d \frac{\pi^2}{90} T^4 . \\ \\ \text{for fermions;} \\ -d \int \frac{d^3k}{(2\pi^3)} T \ln \left(1 + e^{-E(k)/T} \right) = d \frac{7}{8} \frac{\pi^2}{90} T^4 . \end{cases} \tag{2.10}$$

The degeneracy factor of pion arises from the isospin state; we have $d_\pi = 3$ (for both $N_f = 2$ or 3 (Yagi et al., 2005)). This description can be used to describe the system at LHC or ALICE energies, where the pion dominates the medium.

MIT Bag Model

The MIT Bag Model describes the confining of quarks inside the bag, forming a hadrons (Johnson, 1975; DeTar and Donoghue, 1983) where the strong force is described by the empirical bag pressure. At high temperatures and density, the pressure inside the hadron bag becomes very strong, leading the bag to expand and eventually break down. This breakdown is similar to the abrupt change nature of the first-order phase transition, resulting in the hadron bag to de-confine and achieving the de-confinement state, or QGP phase.

Bag Model EoS: The nature of phase transition is dictated by the EoS. Here, the thermal properties of the non-interacting relativistic gases of QGP and pion are connected and serve as a governing EoS,

$$P_{QGP} = d_{QGP}\frac{\pi^2}{90}T^4 - B\,, \tag{2.11}$$

$$\epsilon_{QGP} = 3d_{QGP}\frac{\pi^2}{90}T^4 + B\,, \tag{2.12}$$

$$s_{QGP} = 4d_{QGP}\frac{\pi^2}{90}T^3\,. \tag{2.13}$$

B is the bag parameter whose purpose is similar to the binding energy, $d_{QGP}\left(d_q, d_g\right)$ is the effective degeneracy factor for quarks and gluons. The main feature of the bag model is the strong first order phase transition. In addition, we can use the bag model equation of state to extract the critical temperature for first-order phase transition using the boundary conditions, $P_H\left(T_c\right) = P_{QGP}\left(T_c\right)$.

Lattice QCD

One of the most important features of QCD is asymptotic freedom (Gross and Wilczek, 1973). While, the coupling constant between quarks and gluons in the normal hadron gas condition is very strong, leading to the quark confinement phase. However, when the energy and density of the system increase, the coupling constant between constituents becomes smaller. The quarks and gluons become asymptotically free inside a soup of QGP. The problem arises when one needs to describe the phase transition between these two phases for the EoS from the first principle QCD Lagrangian. Thus, the perturbative approach is not allowed in this case (Politzer, 1973). Furthermore, due to the many-interacting-body problems between the quarks, gluons, and bound state hadrons, the analytic solution for such dynamics is extremely computationally expensive and deemed almost impossible.

However, Ref. (Philipsen, 2006) suggests the numerical technique such that the field variables of the Lagrangian can be defined on a discrete space-time lattice. Despite still being computationally expensive, this method allows us to solve the QCD

Lagrangian through the first principle at assuming thermal equilibrium.

The EoS in lattice QCD can be derived from the grand canonical partition function,

$$Z = \int \mathcal{D}U\mathcal{D}\overline{\psi}\mathcal{D}\psi e^{(S_g - S_f)} \ . \tag{2.14}$$

where the integration is accounting over all U gauge fields (gluons) and $\bar{\psi}, \psi$ fermionic fields (quarks) within SU(3) matrices. S_g represents the gauge action while S_f the fermionic action as the system depends on both gauge and fermionic fields. The expectation values of an observable O can be obtained with

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U\mathcal{D}\overline{\psi}\mathcal{D}\psi O\big(U, \overline{\psi}, \psi\big) e^{S_g - S_f} \ . \tag{2.15}$$

These path integrals are evaluated on a discretized space-time grid using Monte Carlo techniques, i.e., choosing some quark-lattice-sites and gluon-link configurations randomly and evaluating the observables on these fields.

The lattice QCD results help to confirm that at the vanishing baryon chemical potential $\mu_B = 0$, the QCD transition is a crossover type (Aoki et al., 2006; Bazavov et al., 2012). Furthermore, at non-vanishing μ_B although the lattice QCD suffers from the "sign problem" (de Forcrand, 2009), it was reported that as far as lattice QCD can go in the QCD phase diagram, there is no expected critical point from $\mu_B > 0$ until $\mu_B/T \lesssim 3$ (Laermann and Philipsen, 2003; Schmidt, 2006; Bazavov et al., 2019; Borsanyi et al., 2020).

Chiral Model

The Nambu-Jona-Lasinio (NJL) models (Nambu and Jona-Lasinio, 1961; Vogl and Weise, 1991; Klevansky, 1992; Hatsuda and Kunihiro, 1994; Buballa, 2005) and other chiral models (Meisinger and Ogilvie, 1996; Koch, 1997; Meisinger et al., 2002; Meisinger et al., 2004; Fukushima, 2004; Mocsy et al., 2004; Ratti et al., 2006) are theoretical frameworks based on the chiral symmetry. Unlike lattice QCD, the chiral models are effective models aiming to describe the QCD Lagrangian. In the limit where bare quark masses are zero, $m_q \to 0$, QCD Lagrangian is invariant under the $SU\big(N_f\big)_L \times SU\big(N_f\big)_R \times U\big(1\big)_V$ group. However, due to the finite quark masses in nature, this chiral symmetry is explicitly or spontaneously broken already in vacuum, leading to the formation of a quark condensate and the emergence of pseudo-Goldstone bosons, such as pions (Koch, 1997).

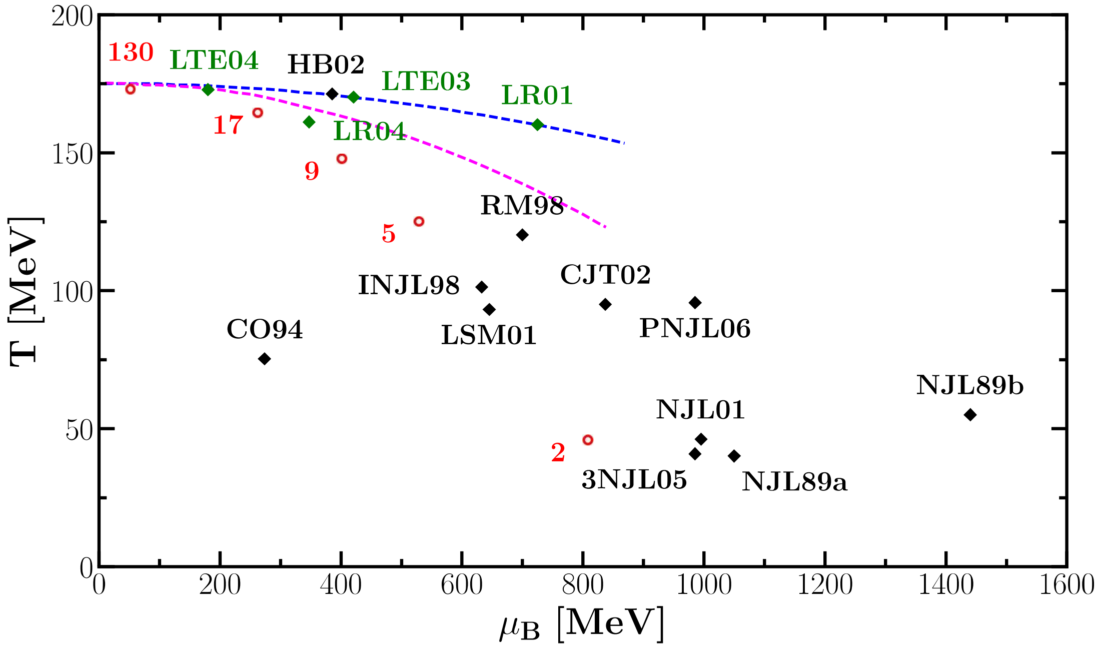

Figure 2.1 The compilation of the predicted location of the QCD critical point from various models, mainly chiral models and lattice QCD (Stephanov, 2006). Black points represent chiral model predictions. Green points indicate lattice predictions. The two dashed lines are the slopes corresponding to $dT/d\mu_B^2$ of the transition line at $\mu_B = 0$. The red circles denote the freeze-out points for heavy ion collisions at corresponding center-of-mass energies in GeV per nucleon.

The thermodynamical properties of the QCD from the chiral models are determined by the path integral of the effective action of the quark condensate field σ and π field. Various phase transition scenarios may be feasible via the chiral models, e.g., a purely crossover or first-order phase transition along with the critical end point.

However, as chiral models are quite model-dependent yet produce powerful predictions, they are typically used in conjunction with other models and experiments as a basis, e.g., implementing with lattice QCD (Borsanyi et al., 2020), or with the hydrodynamics models (Mishustin and Scavenius, 1999; Nahrgang et al., 2011; Herold et al., 2019).

Chiral EoS: The chiral model includes all the sets of baryons and the entire multiplets of scalar, pseudo-scalar, vector, and axial-vector mesons. The grand-canonical potential can then be expressed as

$$
\begin{aligned}
\Omega/V = \mathcal{V}_{\text{meson}} - \mathcal{V}_{\text{vac}} \\
- T\sum_{i\in B} \frac{\gamma_i}{(2\pi)^3}\int d^3k\left[\ln\left(1 + e^{-\frac{1}{T}[E_i^*(k)-\mu_i^*]}\right)\right] \\
+ T\sum_{i\in M} \frac{\gamma_l}{(2\pi)^3}\int d^3k\left[\ln\left(1 + e^{-\frac{1}{T}[E_i^*(k)-\mu_i^*]}\right)\right] .
\end{aligned}
\tag{2.16}
$$

where γ_B, γ_M are the baryonic and mesonic spin-isospin degeneracy factors and $E^*_{B,M}(k) = \sqrt{k^2 + m^{*2}_{B,M}}$ are the single baryon and meson particle energies. The effective baryon chemical potential μ_i^* includes the quark and strange quark chemical potentials. The term $\mathcal{V}_{\text{vac}}$ is the vacuum energy. Here, we include the interaction between baryons and scalar mesons (BM), the vector mesons ($\mathcal{L}_{\text{vec}}$) and the scalar self-interactions $\mathcal{L}_0$. The scalar meson interaction induces spontaneous symmetry breaking of the chiral symmetry ($\mathcal{L}_{\text{SB}}$). The effective mesonic potential can be written by $\mathcal{V}_{\text{meson}} = -\mathcal{L}_{\text{vec}} - \mathcal{L}_0 - \mathcal{L}_{\text{SB}}$. Figure 2.1 shows potential critical points have been reported according to chiral models and lattice calculations, where a detailed derivation of the chiral equation of state can be found in (Omana Kuttan et al., 2022; Steinheimer et al., 2022; Omana Kuttan et al., 2023).

2.1.2 Beam Energy Scan and Low Energy Regime

The Beam Energy Scan (BES) program (Collaboration, 2014; Luo, 2016; Bzdak et al., 2020) plays an important role in the QCD phase diagram exploration. Heavy-ion will be collided at various energy ranges, scanning all over the QCD phase diagram, aiming to pinpoint the location of a critical point, if it exists. This critical point is expected to be located in the environments created by low- to intermediate-energy heavy-ion collisions. The corresponding trajectories in the QCD phase diagram are shown in Figure 2.2.

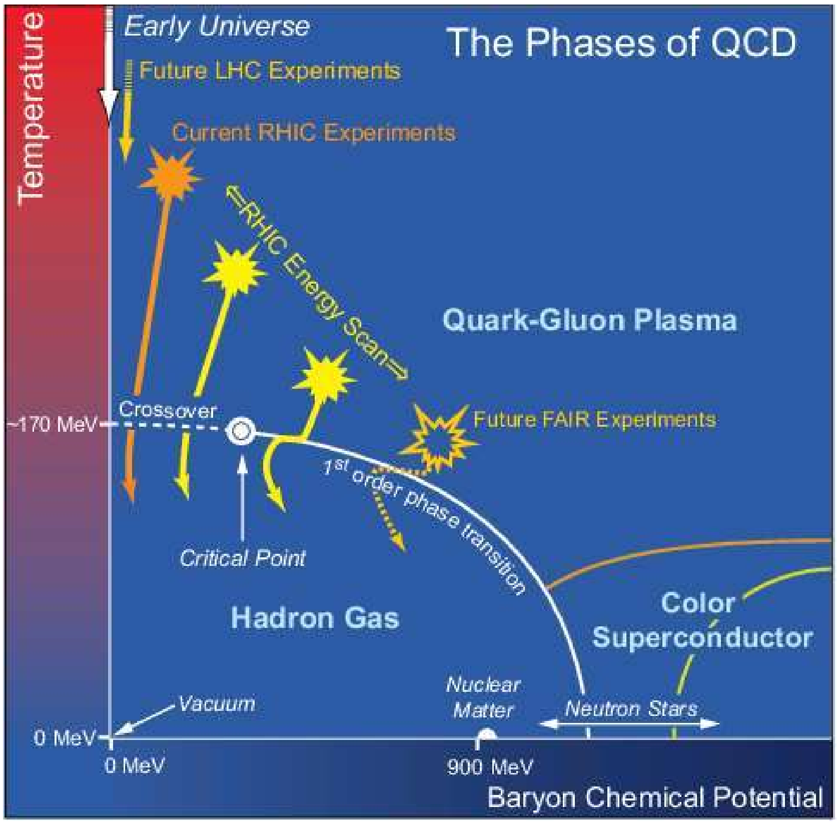

Figure 2.2 QCD diagram with BES program and various facilities (Collaboration, 2014).

- **Relativistic Heavy-ion Collider (RHIC):** The facility is originally designed for exploring the QCD at very high energies particularly searching for the existence of the new phases of QCD matter, QGP. After many updates, it now can provide a broader range of collision energies from $7.7 - 200$ GeV. This allows RHIC to initiate the Beam Energy Scan (BES) program (Luo, 2016; Bzdak et al., 2020; Collaboration, 2014) which most of the heavy-ion collision facilities will collectively join with the aims for locating the critical point and extracting the EoS as well as new physics at extremely dense medium.

- **Future Facilities (FAIR and HADES):** The upcoming Facility for Antiproton and

Ion Research (FAIR) at GSI will also join the BES effort (Almaalol et al., 2022), offering a new dimension to the study. FAIR's energy and intensity (Friman et al., 2011; Ablyazimov et al., 2017; Durante et al., 2019; Bzdak et al., 2020) allow for a boarder range of physics study from nuclear physics to astrophysics. In terms of heavy-ion physics research, FAIR will extend the BES coverage to even lower energy ranges $\sqrt{s_{NN}} < 7.7$ GeV and more dense medium allowing to probe the critical behavior from first-order phase transition, the formation of rare probes like exotic nuclei and ultimately the EoS (for the neutron stars). HADES (High Acceptance Di-Electron Spectrometer) is also one of the facilities at GSI focusing on the low energy and dense medium similar to FAIR. It specializes in the study the rare probes such as dileptons and exotic nuclei. Dileptons are a powerful probe for the properties of the QCD matter at the early stage. While the (exotic) (hyper)nuclei serve as valuable basis for the EoS of neutron star conditions (Weber, 2005; Lattimer, 2021; Most et al., 2023). Furthermore, as we will discuss in Ch. VIII, the pion-induced reactions at HADES also provide a unique opportunity to study these rare probes at smaller system sizes.

2.2 Space-Time Evolution

One should note the critical importance of understanding the space-time picture of heavy-ion collisions and their evolution (Shuryak, 1978). To comprehend the physical signatures (observables) produced in these collisions, we need a detailed understanding of the medium's evolution after the impact until the particle free streaming into the detectors. Figure 2.3 depicts this sequence of events.

When the two Lorentz contracted heavy nuclei collide, a huge amount of energy and momentum exchange occurs. At the overlapping region, an extremely hot, dense, and chaotic system is created. This medium is hardly in any equilibrium since the change is strong and sudden. Numbers of theoretical models (Bravina et al., 1999; Mishra et al., 2008; Sorensen, 2010a; Sorensen, 2010b; Bally et al., 2022) is trying to describe this pre-equilibrium stage since realistic initial conditions are essential to any dynamical evolution. Nevertheless, a clear understanding is still a work in progress.

After some time, this initial chaos settles down, the system approaches a state of (local) thermal equilibrium allowing for the hydrodynamics description (Yagi et al., 2005). The matter at this stage is in the QGP state which exhibits properties

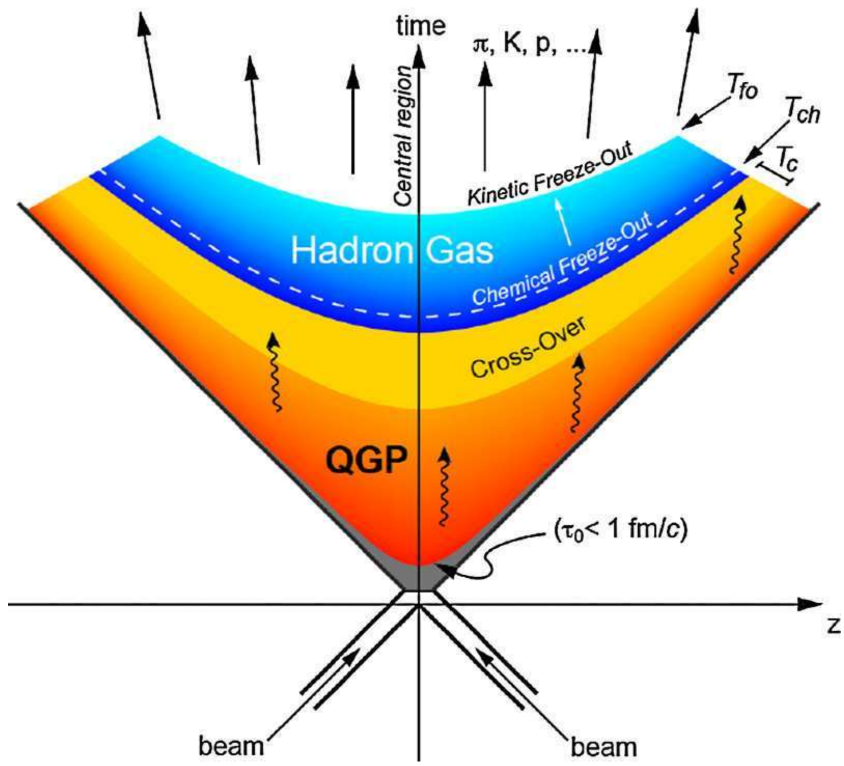

Figure 2.3 Space-Time Evolution (Braun-Munzinger and Dönigus, 2019).

remarkably similar to a perfect fluid due to its high density and asymptotic freedom of strong interactions between its constituents (Teaney, 2010; Adamczyk et al., 2017).

As the energy density of the medium drops below a critical point, the system undergoes a phase transition producing a large number of hadrons or fragmentation occurs (Buyukcizmeci et al., 2020; Botvina et al., 2021; Botvina et al., 2022; Buyukcizmeci et al., 2023). Until all the partons are bound into hadrons, then the system is considered entering the hadronic stage. The newly formed hadrons continue to interact and scatter with each other leading to continuous destruction and creation of new particles. When a balance in the number (density) of different particle types is achieved, we call this stage a chemical equilibrium or chemical freeze-out.

Until the system is cool and big enough, these hadrons become too sparse to interact further. Finally, at the last stage of the space-time evolution, the hadrons essentially “freeze” in their momentum states and propagate toward the detectors. This stage is called kinetic freeze-out.

Various theoretical models have already succeeded in describing the dynamics during and after fragmentation/hadronization. Some models, like the thermal models (Andronic, 2014; Andronic et al., 2018), focus on the macroscopic properties of the system, treating hadrons as relativistic gases (at chemical freeze-out). While some other models, like transport models (Wolter et al., 2022), treat the dynamics of individual hadron interactions at the microscopic picture.

The dynamics within each stage of this space-time evolution can be linked to the EoS as it dictates the expansion and cooling rate as well as the critical behavior of the system. However, the EoS alone could not directly imply a complete set of final stage observables. The influence from the initial stage has to be considered (Mishra et al., 2008; Sorensen, 2010a; Sorensen, 2010b; Bally et al., 2022) as well as the interplay between various factors during the evolution. These factors ultimately shape the final-state observables, e.g., collective flows (Ollitrault, 1992; Reichert et al., 2022), fluctuations (Skokov et al., 2013; Adare et al., 2016; Luo and Xu, 2017), and etc (Iancu, 2014; Vovchenko et al., 2020; Acharya et al., 2022).