CHAPTER VII
INVESTIGATING CLUSTER PRODUCTION MECHANISMS

7.1 Thermal vs Coalescence

(Hyper)(Light)Nuclei have been highlighted as crucial tools for investigating the medium properties and serving as a basis for studying the EoS around neutron star density regimes. In the Ch. IV, we have explored and discussed the roles of the source geometry and the cluster formations in the context of space-time exploration. We further emphasized on the different space-time pictures of various formation mechanisms in Ch. V especially two common approaches, i.e., the thermal model and the coalescence model. Despite the fact that their underlying physics assumptions differ significantly, there is no clear evidence to determine which mechanisms are really responsible for the cluster formation as both models result in similar estimated particle yields across a broad spectrum of collision energies.

The thermal model describes the particle production through the thermodynamic properties of the fireball, i.e., temperature and chemical potential, and typically within the grand canonical ensemble. It presumes that particle yields are fixed after the medium is fully thermalized at chemical freeze-out, thus implying that all particles form at a fixed temperature $T_{\mathrm{chem}} \gtrsim 70$ MeV and corresponding chemical potential $\mu_{\mathrm{B,chem}}(T)$ (Andronic et al., 2018). This raises questions about the survival of light clusters like the deuteron, with a binding energy of just a few MeV, in such an environment and contrasts with the deuteron bottleneck concept in Big Bang Nucleosynthesis (Pospelov and Pradler, 2010), where cluster formation requires cooling to match deuteron binding energies. Despite these challenges, the thermal model remains widely used to estimate particle yields (Vovchenko et al., 2020), even as it is difficult to incorporate dependencies like wavefunctions which are crucial at lower energies where the internal structure of clusters matters (Juric et al., 1973; Abelev et al., 2010; Adam et al., 2016; Dönigus, 2013; Andronic et al., 2018; Blaschke et al., 2020).

On the other hand, although the coalescence model is also based on statistical mechanics, it is closer in spirit to the microcanonical ensemble, allowing for the study of medium dynamics. Here, coalescence occurs at the kinetic freeze-out;

if two or more free nucleons, after their final collisions and decays, are close enough in phase-space, they will form a cluster. Typically, the temperature and volume of the source size are smaller than in the thermal model making it possible for the clusters to survive. In contrast to the thermal model's limitations, the coalescence model can accommodate various other factors and dynamical considerations. Distinguishing between thermal and coalescence methods for cluster production becomes crucial.

In this chapter, we use these distinctions to investigate which mechanism is realized in nature for cluster formation in heavy-ion collisions. In the thermal model, occurring at chemical freeze-out, all hadrons, including clusters and resonances, are generated at the same time before undergoing any decay processes. Consequently, the final yields of clusters may not experience any fluctuations due to the stochastic nature of the decays or uncorrelated. Conversely, the coalescence model operates at kinetic freeze-out, occurring after all resonances have decayed, potentially allowing for the effects of event-by-event fluctuations from resonance decays to influence the final yields.

In our study, we specifically consider isospin fluctuations. Although the thermal model adheres to conservation laws, it typically employs grand canonical ensembles[*] (Cleymans and Satz, 1993; Becattini et al., 1998; Florkowski et al., 2002; Cleymans et al., 2006; Andronic et al., 2011; Petrán et al., 2013; Vovchenko et al., 2016; Andronic et al., 2019). This implies that while the thermal model can indeed capture isospin fluctuations, it can only provide averaged values derived from these fluctuations (Vovchenko and Stoecker, 2019). Therefore, we will examine if there is any correlation between isospin fluctuations and light cluster yields in the coalescence approach, knowing that the thermal model will always yield uncorrelated results.

7.2 Isospin triggering

Due to isospin conservation, the number (density) of nucleons is correlated with the emitted charged pions at the kinetic freeze-out through Δ decays (Reichert et al., 2019; Reichert et al., 2021). While other charged particles, e.g. kaons, also carry isospin, the freeze-out nucleons and charged pions are still the dominant species at all

[*]Even for the canonical ensemble (Vovchenko et al., 2018a), the total baryon number and isospin are also fixed (for a $b = 0$ collision they are identical to the initial state). The light nuclei, e.g., deuterons with isospin zero, are therefore not correlated with the pion isospin fluctuations. The main driver of pion isospin fluctuations are resonance decays after the fixing of the deuteron (and other cluster) numbers

energies. The correlation arises from the isospin content exchanges via,

$$p_{part} \rightarrow n_{fr} + \pi^{+}$$
$$n_{part} \rightarrow p_{fr} + \pi^{-} ,$$

where p_{part} and n_{part} represent participating protons and nucleons, respectively, while p_{fr} and n_{fr} denote protons and nucleons at kinetic freeze-out. It's important to note that we assume a fixed volume where the participants $A_{part} = N_{Au} + Z_{Au}$ and the isospin ratio $N_{part}/Z_{part} = N_{Au}/Z_{Au}$ do not fluctuate to demonstrate a clear isospin fluctuation effect (Kittiratpattana et al., 2022). With this assumption, we can estimate the number of light clusters using coalescence models.

To illustrate our argument, consider an initial scenario with head on Au+Au collisions with $p_{part} = 2 \times Z_{Au} = 2 \times 79 = 158$ participating protons. If we trigger on an extreme event with 158 emitted π^{+} and zero π^{-}, we would have a pure $n_{fr} = 158$ medium at kinetic freeze-out. According to the coalescence model, the probability of forming other light nuclei should vanish in this scenario. Conversely, if we trigger on an event with pure π^{-} and p_{fr}, there should be no light nuclei present. Therefore, we can deduce that the yields of light nuclei can be expressed in terms of the relative difference in charged pion yields $\Delta Y_{\pi} \equiv \left(Y_{\pi^-} - Y_{\pi^+}\right)$ which should exhibit a distinct maximum.

In summary, the presence of a local maximum in the deuteron yield at a fixed (or tightly constrained) A_{part} as a function of ΔY_{π} serves as a distinguishing factor between thermal deuteron production and the coalescence approach. Additionally, we will also study higher mass clusters to validate this scenario.

7.2.1 Simple estimates

The simple coalescence model states that,

$$d = \tilde{B}_2 \cdot n_{fr} \cdot p_{fr} , \tag{7.1}$$
$$t = \tilde{B}_3 \cdot n_{fr}^2 \cdot p_{fr} , \tag{7.2}$$
$$^3He = \tilde{B}_3 \cdot n_{fr} \cdot p_{fr}^2 \tag{7.3}$$

From the previous discussion, the total number of neutrons at kinetic freeze-out, denoted as n_{fr}, is determined by the sum of initial neutrons from the nuclei, $2N_{Au}$, added to the number of π^- responsible for newly produced neutrons, and then subtracted by the number of π^+ responsible for converting initial participant neutrons into protons at kinetic freeze-out. Similarly, the total number of protons at kinetic freeze-out, denoted as p_{fr}, can be expressed similarly. We write n_{fr} and p_{fr} in terms of the relative difference in charged pions ΔY_π,

$$n_{fr} = 2N_{Au} - \Delta Y_\pi \,, \tag{7.4}$$

$$p_{fr} = 2Z_{Au} + \Delta Y_\pi \,. \tag{7.5}$$

Then, we can estimate the yields of light clusters based on the coalescence model, which is expressed as:

$$d = \widetilde{B}_2 \cdot \left(2N_{Au} - \Delta Y_\pi\right) \cdot \left(2Z_{Au} + \Delta Y_\pi\right) \tag{7.6}$$

$$t = \widetilde{B}_3 \cdot \left(2N_{Au} - \Delta Y_\pi\right)^2 \cdot \left(2Z_{Au} + \Delta Y_\pi\right) \tag{7.7}$$

$$^3He = \widetilde{B}_3 \cdot \left(2N_{Au} - \Delta Y_\pi\right) \cdot \left(2Z_{Au} + \Delta Y_\pi\right)^2 \,, \tag{7.8}$$

where $\widetilde{B}_A$ is a coalescence factor from (Kittiratpattana et al., 2022) which has been discussed already in Ch. VI.

The results from Eq. (7.6)-(7.8) are shown in Fig. 7.1. The estimated yields of deuterons (d) are represented by the solid pink line, tritons (t) by the blue dashed line, and 3-Helium (^{3}He) by the orange dotted line. As anticipated, all three show a district maximum with respect to the ΔY_π triggering. The local maximum of deuterons occurs at $\Delta Y_\pi = 39$. This arises from the fact that deuterons, consisting of one proton and one neutron, will have the highest probability to form when the medium is dominated by an equal number of protons and neutrons at kinetic freeze-out, i.e., $\Delta Y_\pi = N_{Au} - Z_{Au} = 39$.

For the tritons and ^{3}He, their respective local maxima at $\Delta Y_\pi = \frac{1}{3}\left(2N_{Au} - 4Z_{Au}\right) = -\frac{80}{3}$ for tritons and $\Delta Y_\pi = \frac{1}{3}\left(4N_{Au} - 2Z_{Au}\right) = \frac{314}{3}$. Additionally, at $\Delta Y_\pi = 39$, where the proton and neutron content in the medium are evenly distributed coupled with the symmetry between tritons and ^{3}He, we expect their yields to be equal.

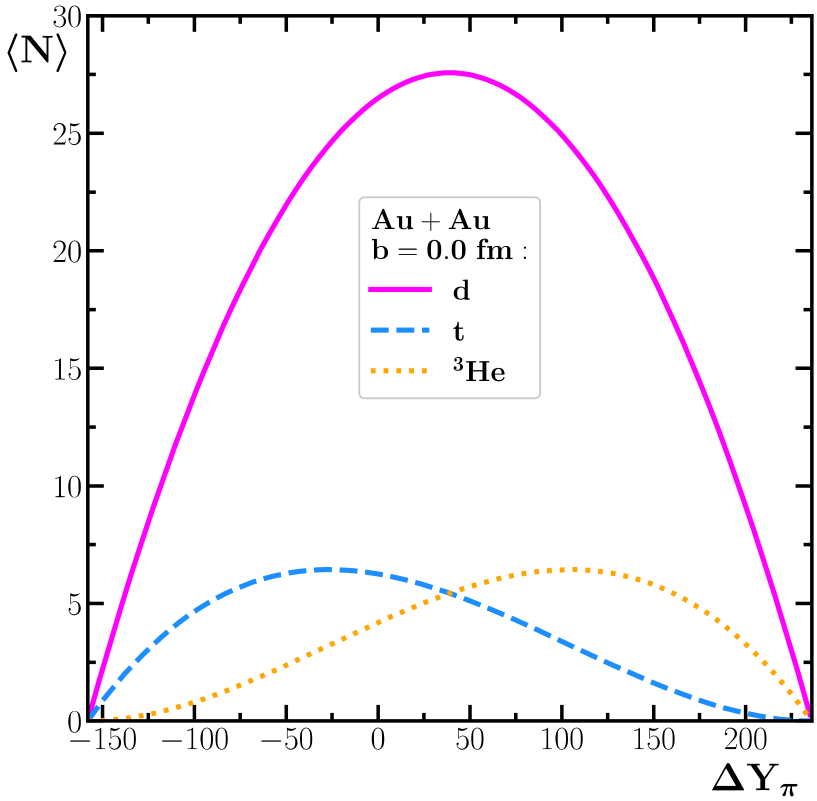

Figure 7.1 The theoretical estimation of the deuteron d (pink full line), triton t (blue dashed line), and ^{3}He (orange dotted line) production according to the Eq. (7.6)- (7.8) for central Au+Au reactions as a function of ΔY_π.

7.3 Qualitative Estimates

For the simplified theoretical model estimates, we assumed that the number of participants as well as their isospin N/Z ratio do not fluctuate, i.e., a fixed volume and $N/Z = N_{Au}/Z_{Au}$ for the estimation of the deuteron (and higher mass cluster) yields. However, this is not the case for a realistic situation. Thus, in this section, we will test our toy model with a detailed microscopic simulation of the UrQMD model which does not have such assumptions (even at the most central collisions). The results show nearly the same behavior as our simplified toy model.

In UrQMD model (Bass et al., 1998; Bleicher et al., 1999; Bleicher and Bratkovskaya, 2022) version v3.5, light clusters are produced by phase space coalescence from nucleons at kinetic freeze-out (see also (Sombun et al., 2019; Hillmann et al., 2022; Kireyeu et al., 2022) for details).

We focus on central Au+Au collisions with a center-of-mass energy range of $\sqrt{s_{NN}} = 3 - 8$ GeV, optimal for testing our concept, as pions and participating nucleons are strongly correlated, with their numbers being nearly equal or at least in the same order of magnitude. Moreover, at this energy range, both pions and nucleons dominate the medium, and when the isospin is (or is trying to become) equilibrated, the effects of isospin fluctuations on pions and nucleon numbers are more apparent than at higher energies. Although, at higher energies, the net charged pion fluctuations are stronger, it is compensated by the lesser deuteron yields and less correlation with the nucleons, as various hadrons also participate in the isospin exchange.

7.3.1 Freeze-out time distributions

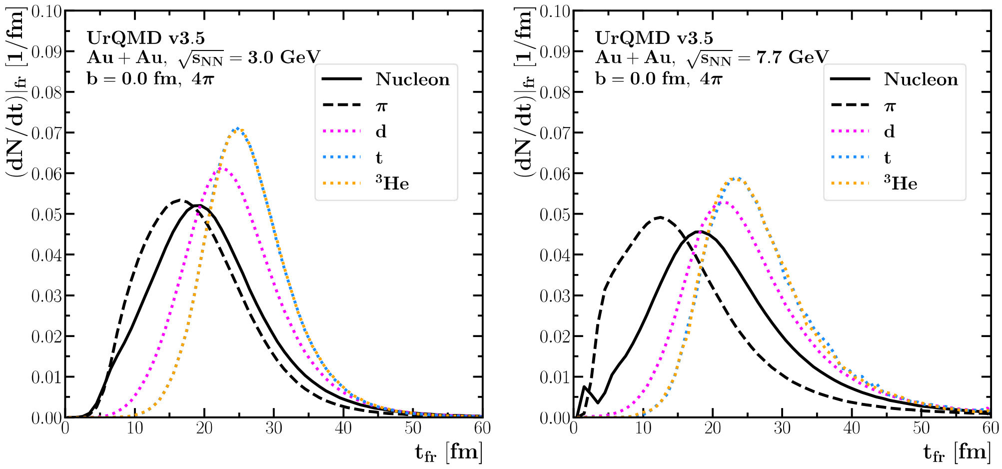

Figure 7.2 Freeze-out time distribution of nucleons (full black line), pions (dashed black line), deuterons (dotted pink line), tritons (dotted blue line), and 3He (dotted orange line).

To begin and illustrate the idea, Fig. 7.2 shows the normalized freeze-out time distribution of the nucleons (solid black line), pions (dashed black line), deuterons d (dotted pink line), tritons t (dotted blue line) and 3-Helium ^{3}He (dotted orange line) in very central Au+Au collisions at $\sqrt{s_{NN}} = 3$ GeV (left panel) and at $\sqrt{s_{NN}} = 7.7$ GeV (right panel).

It is evident that light nuclei freeze-out after the pions have decoupled from the system, i.e., after isospin equilibration, as expected. Therefore, isospin fluctuations can indeed influence cluster formation. Although at both energies pions appear to decouple shortly before nucleons, this process is faster at higher energies ($\sqrt{s_{NN}} =$

7.7 GeV). This is due to the fact that, unlike our toy model, in the more realistic scenarios simulated by UrQMD, other hadrons can still be emitted together with the nucleons after the pions have decoupled. However, the overall assumption still holds true, as pions decouple before the clusters.

7.4 Light cluster yields versus isospin fluctuation

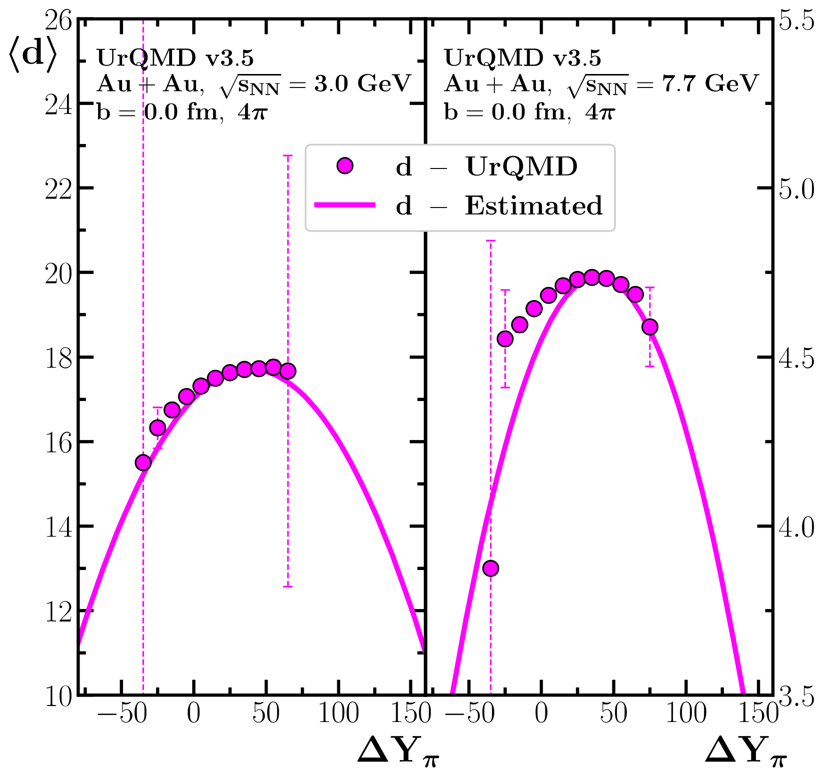

Figure 7.3 Deuteron yield as a function of ΔY_π for Au+Au reactions. The UrQMD results are shown by red circles. The estimated yield, Eq. (7.7), is represented by the full red line. Left: Results at $\sqrt{s_{\mathrm{NN}}} = 3$ GeV. Right: Results at $\sqrt{s_{\mathrm{NN}}} = 7.7$ GeV.

Finally, we contrast the estimated deuteron yields from the toy model with deuteron yields obtained from UrQMD simulations as a function of relative charged pion difference ΔY_π. The comparison at $\sqrt{s_{\mathrm{NN}}} = 3.0$ GeV is depicted in Fig. 7.3 (left panel), and at $\sqrt{s_{\mathrm{NN}}} = 7.7$ GeV in Fig. 7.3 (right panel). We clearly observe a local maximum in the deuteron yields at both energies around $\Delta Y_\pi = 39$, consistent with the expectation from the toy model.

The same comparison on light clusters with $A = 3$ is done and shown

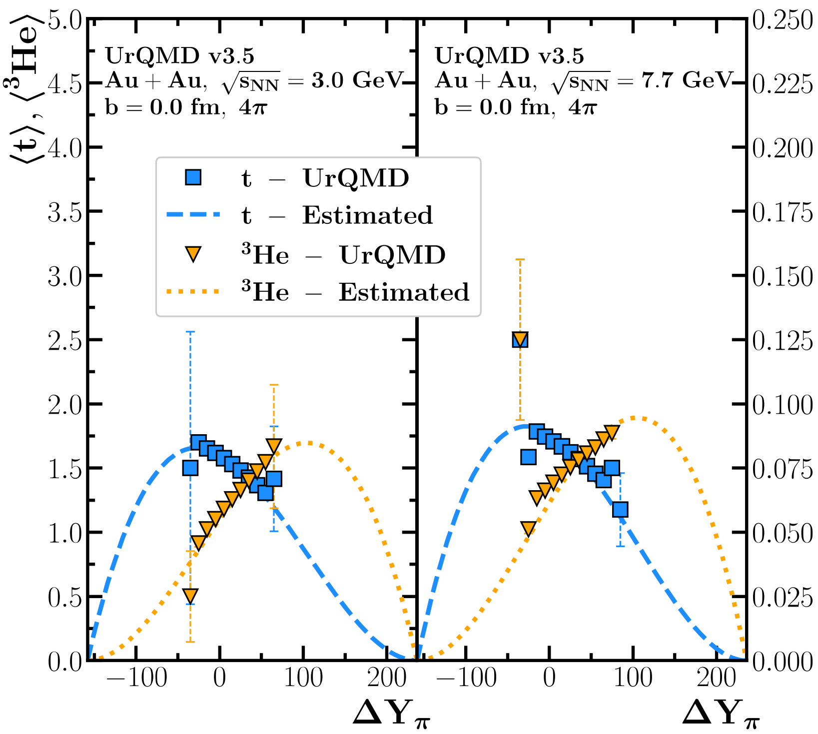

Figure 7.4 The ΔY_π dependent of triton (blue squares and dashed blue line) and ^{3}He (orange triangles and dotted orange line) yields. The UrQMD results are shown by symbols. The estimated yields, Eqs. (7.7) and (7.8), are represented by the lines. Left: Results at $\sqrt{s_{NN}} = 3$ GeV. Right: Results at $\sqrt{s_{NN}} = 7.7$ GeV

in Fig. 7.4, depicting the simulated triton (blue crosses) and ^{3}He (orange triangles) yields from UrQMD at $\sqrt{s_{NN}} = 3$ GeV (left panel) and $\sqrt{s_{NN}} = 7.7$ GeV (right panel) with the corresponding estimated yields from the toy model represented by the blue dashed and orange dotted line, respectively. We observe that both the UrQMD simulated tritons and ^{3}He at both energies follow the estimated yields. Especially at $\sqrt{s_{NN}} = 3.0$ GeV, where the tritons exhibit the maximum yield at $\Delta Y_\pi = -26.67$, we observe a close resemblance between UrQMD and estimated values.

Finally, we provide the energy dependence of the cluster yields as a function of ΔY_π from $E_{lab} = 1.23A - 40A$ GeV. the cluster yields are normalized by their respective yields at $\Delta Y_\pi = 39$. We can clearly observe that the local maxima of the deuteron yields and the $A = 3$ clusters are present and independent of the beam energy.

Also, some deviation from the UrQMD simulation to the toy model is observed as the distributions of light nuclei become broader along with the energies. This

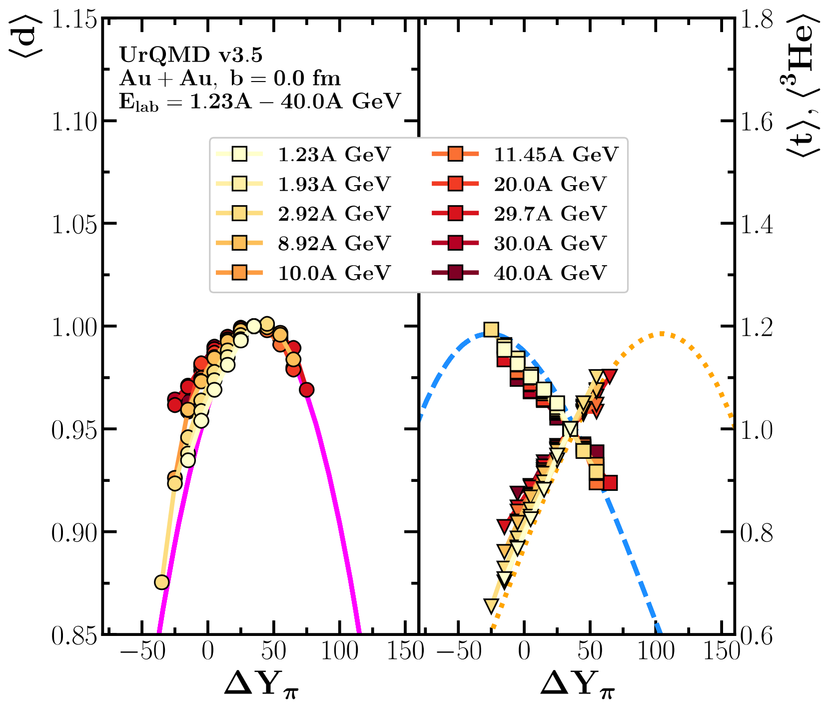

Figure 7.5 Distribution of cluster yields on the ΔY_π spectrum is normalized to unity at $\Delta Y_\pi = 39$. The symbols represent simulation results from various collision energies ranging from $E_{lab} = 1.23A$ GeV to $E_{lab} = 40A$ GeV in ultra-central Au+Au reactions from UrQMD. Left: Deuteron distribution. Right: Triton and ^{3}He distribution.

deviation can be attributed to the fact that in the toy model, isospin equilibration is assumed only between pions and nucleons. In a realistic scenario, some isospin is also carried by other hadron species such as charged kaons, Σ, etc.

We conclude our chapter with the observation that the coalescence model exhibits an energy-independent local maxima in cluster yields, e.g., the deuteron yields at $\Delta Y_\pi = 39$ (for Au+Au reactions). This distinct dependence on the isospin triggering allows us to potentially resolve tensions between the thermal model and the coalescence model. Since the thermal model usually uses grand canonical ensembles, it does not show any dependence of the cluster yield on ΔY_π. This is because the isospin fluctuations occur before kinetic freeze-out inducing the correlation between cluster yields and the emitted charged pions (and nucleons). In the thermal model, all hadron yields are generated or emitted simultaneously at the chemical freeze-out. Consequently, the cluster yields from the thermal model do not correlate with any isospin fluctuations, or at most, have been accounted for only on an averaged basis.

Our studies indicate that the cluster formation is governed by coalescence at the kinetic freeze-out rather than a direct emission from the chemical freeze-out by thermal productions. This approach can be measured in any ultra-central Au+Au collision facilities at $\sqrt{s_{NN}} = 3 - 8$ GeV.